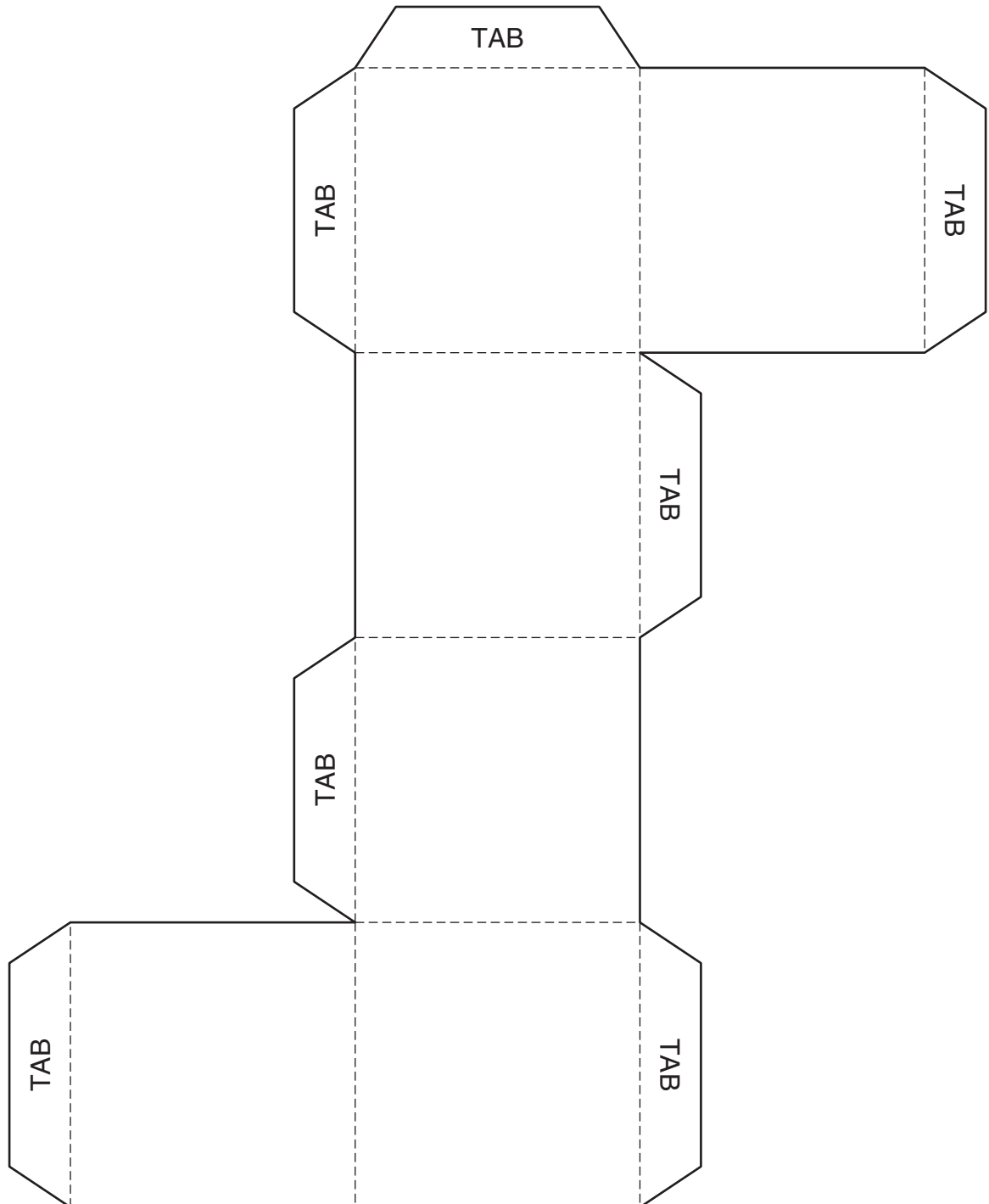


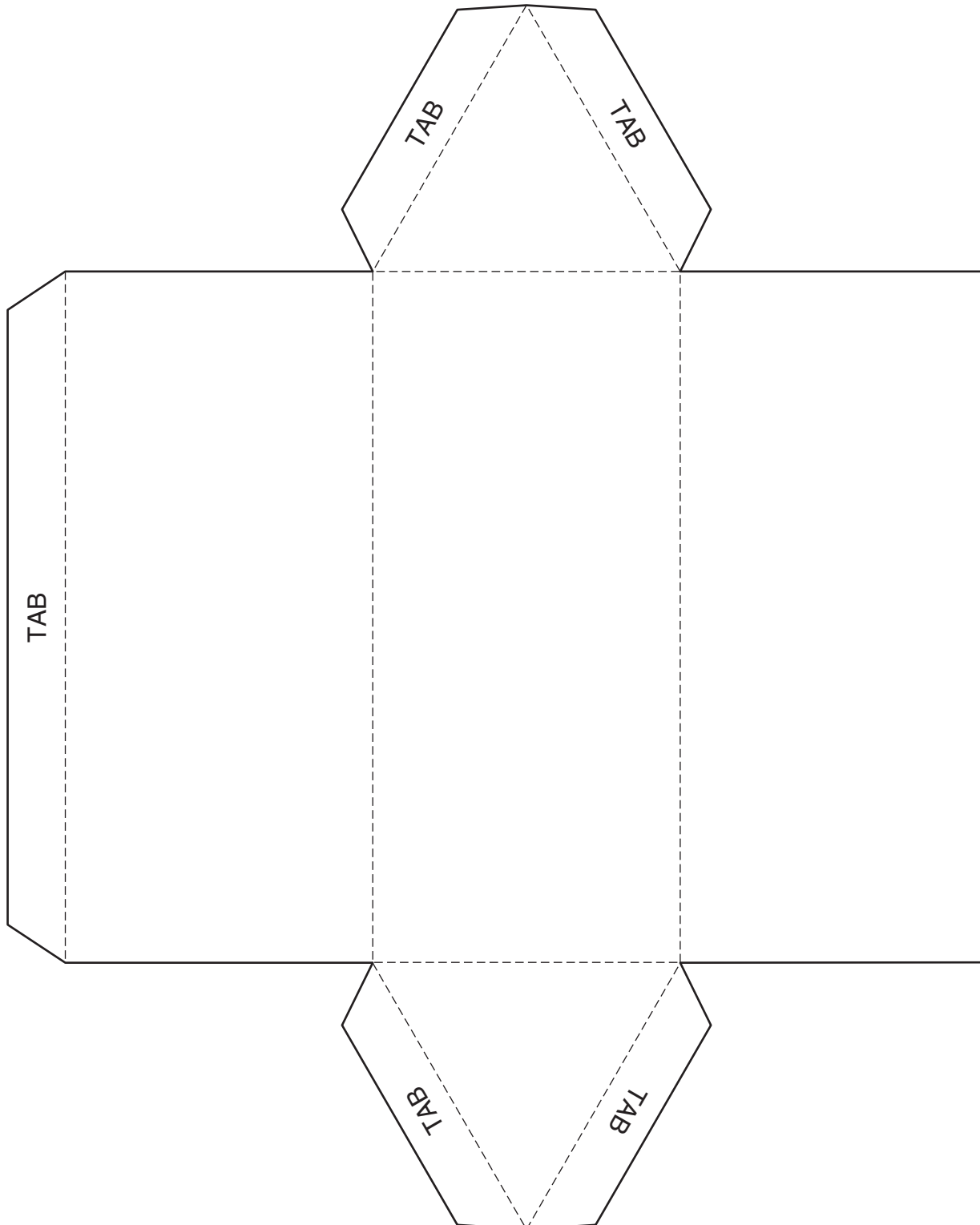
LESSON
11•1**Cube Pattern**

1. Cut on the solid lines.
2. Fold on the dashed lines.
3. Tape or glue the tabs inside or outside the shape.



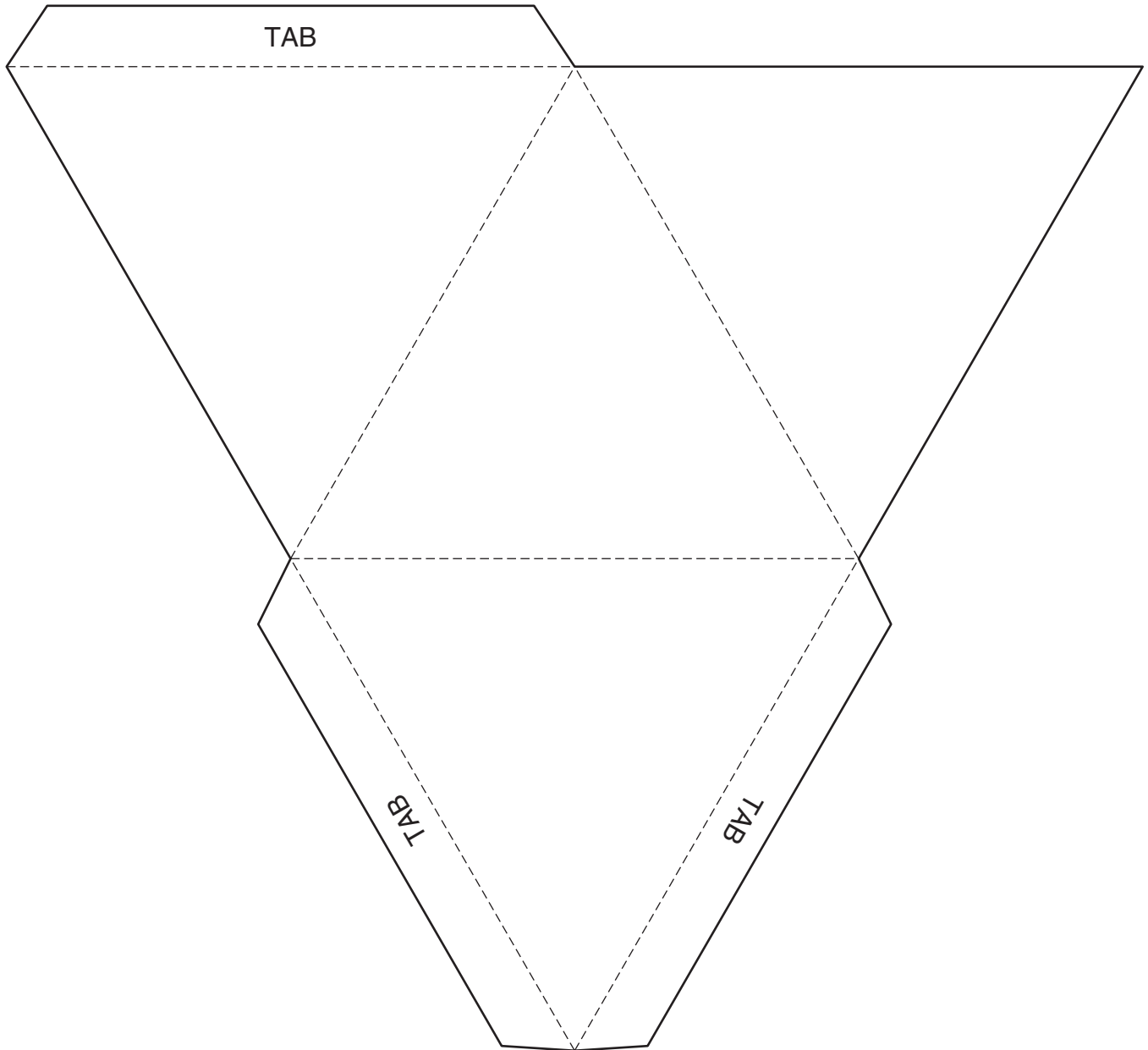
LESSON
11•1**Triangular Prism Pattern**

1. Cut on the solid lines.
2. Fold on the dashed lines.
3. Tape or glue the tabs inside or outside the shape.



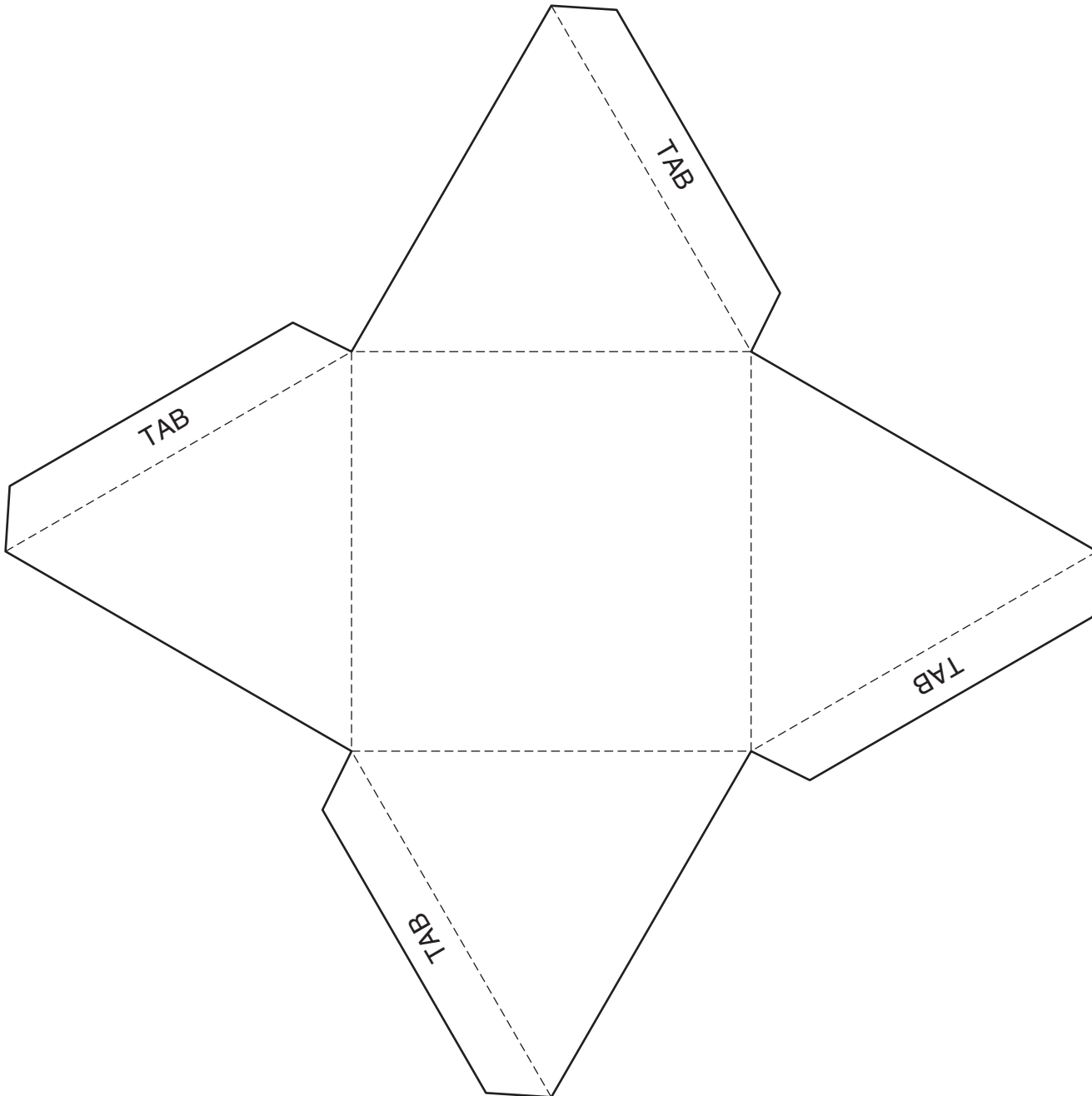
LESSON
11.1**Triangular Pyramid Pattern**

1. Cut on the solid lines.
2. Fold on the dashed lines.
3. Tape or glue the tabs inside or outside the shape.



LESSON
11•1**Square Pyramid Pattern**

1. Cut on the solid lines.
2. Fold on the dashed lines.
3. Tape or glue the tabs inside or outside the shape.



STUDY LINK
11•1

Cube Patterns



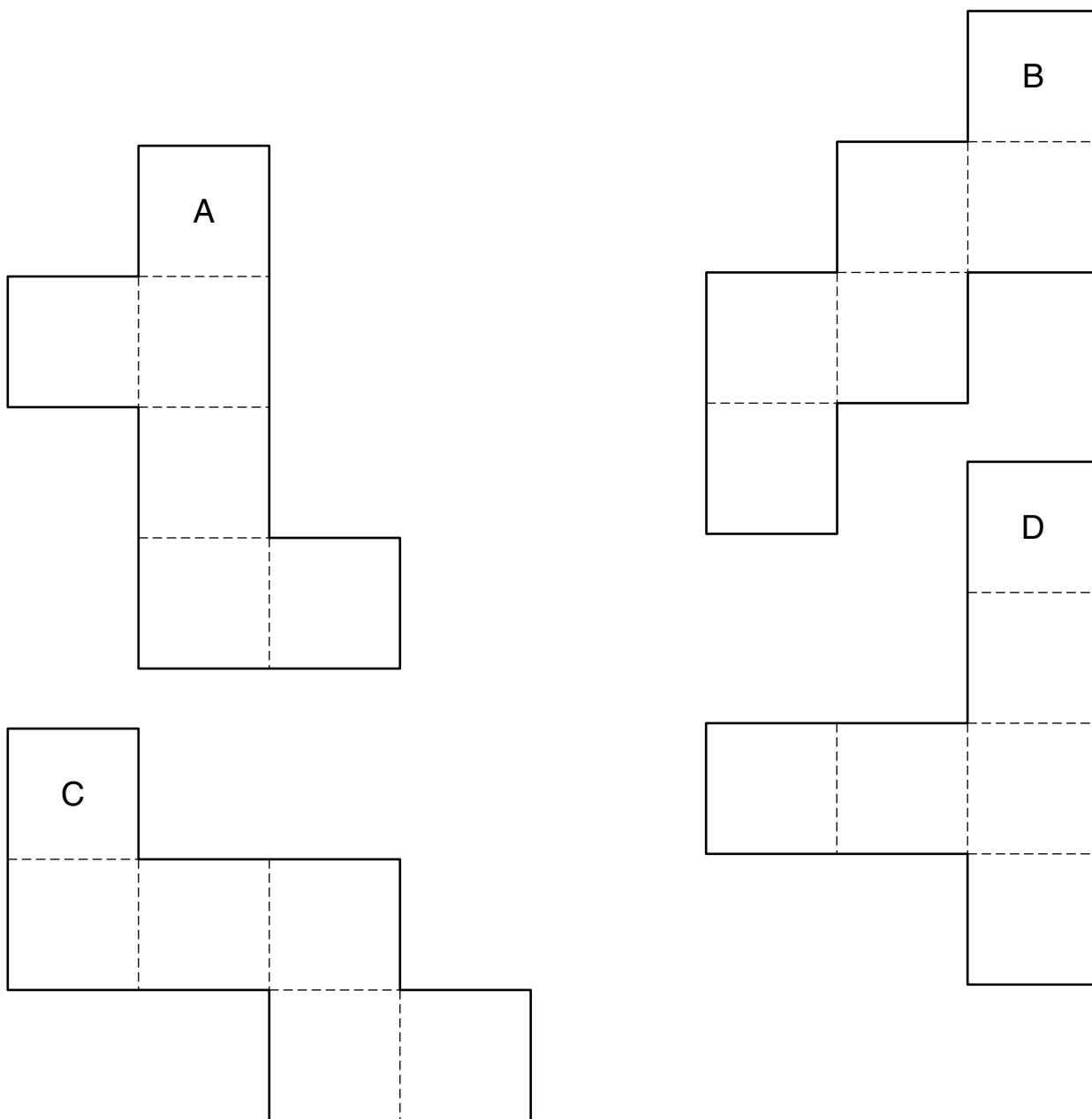
There are four patterns below. Three of the patterns can be folded to form a cube.

1. Guess which one of the patterns below cannot be folded into a cube.

My guess: Pattern _____ (A, B, C, or D) cannot be folded into a cube.

2. Cut on the solid lines, and fold the pattern on the dashed lines to check your guess. Did you make the correct guess? If not, try other patterns until you find the one that does not form a cube.

My answer: Pattern _____ (A, B, C, or D) cannot be folded into a cube.



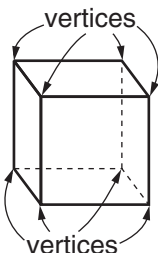
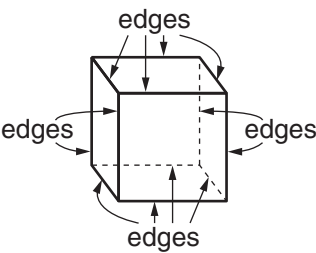
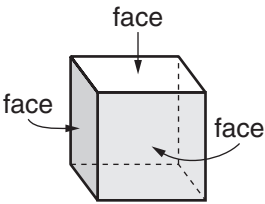
LESSON

11•1

Exploring Faces, Vertices, and Edges



- ◆ A flat surface of a geometric solid is called a face.
- ◆ A corner of a geometric solid is called a vertex. The plural of vertex is vertices.
- ◆ An edge of a geometric solid is a line segment or curve where two surfaces meet.

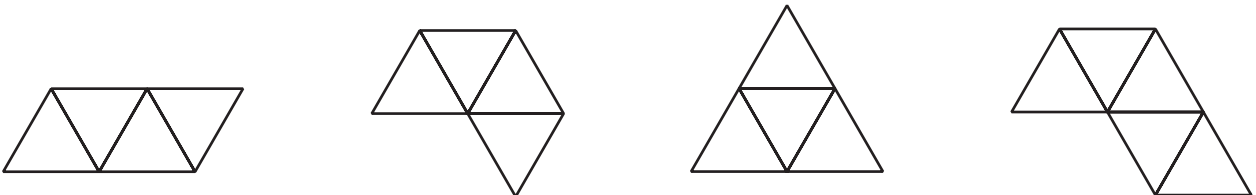


1. Complete the table.

Polyhedron	Faces	Vertices	Faces + Vertices	Edges
Cube	6	8	$6 + 8 = 14$	12
Tetrahedron				
Octahedron				
Dodecahedron				
Icosahedron				

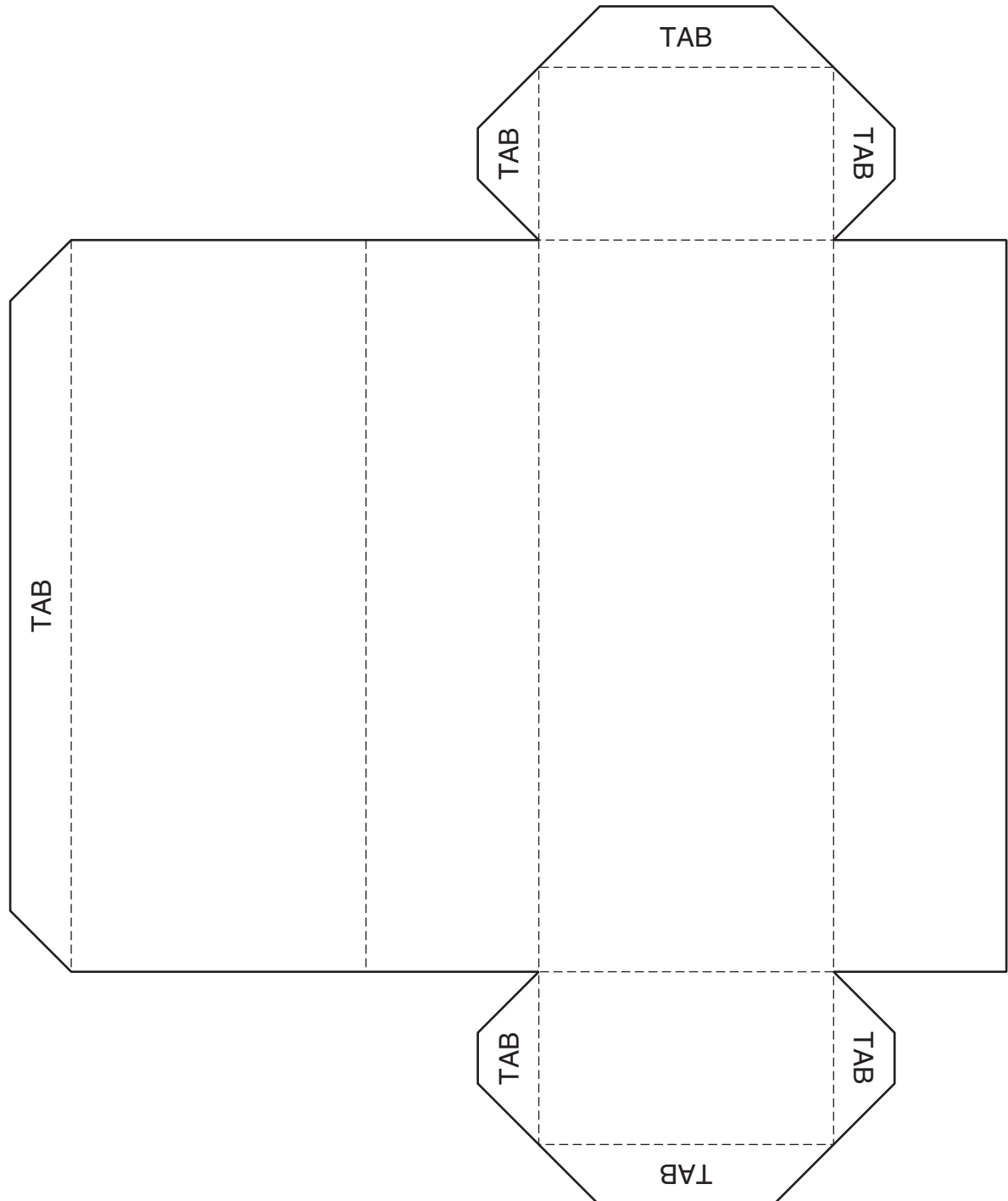
2. Compare the values in the Faces + Vertices column with the Edges column. What do you notice?

3. Two of the patterns below can be folded to make a tetrahedron. Cross out the patterns that will not make a tetrahedron. Circle the patterns that will make a tetrahedron. Explain your solution strategy.



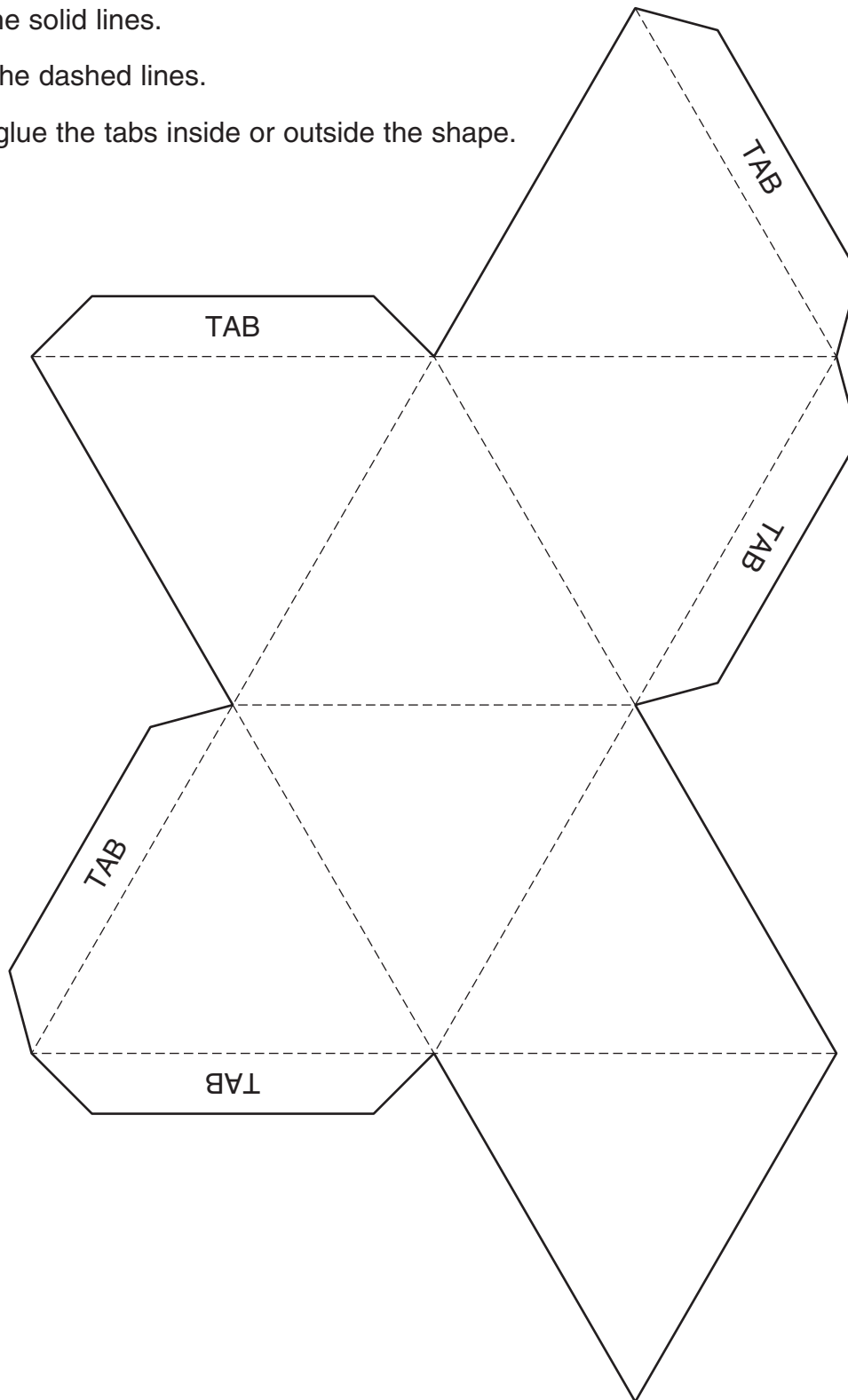
LESSON
11.1**Rectangular Prism Pattern**

1. Cut on the solid lines.
2. Fold on the dashed lines.
3. Tape or glue the tabs inside or outside the shape.



LESSON
11•1**Octahedron Pattern**

1. Cut on the solid lines.
2. Fold on the dashed lines.
3. Tape or glue the tabs inside or outside the shape.

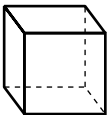
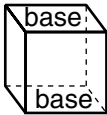
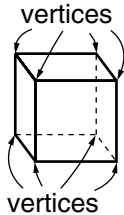
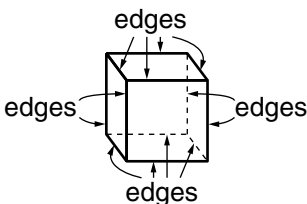
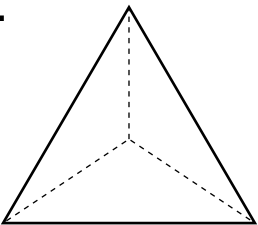
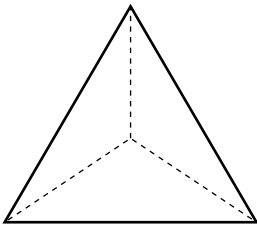
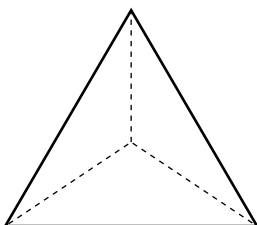
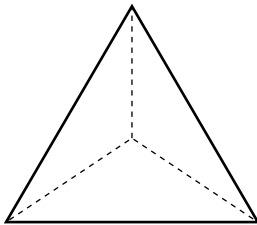
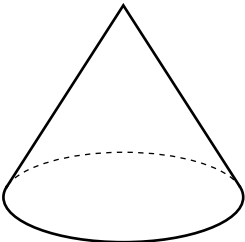
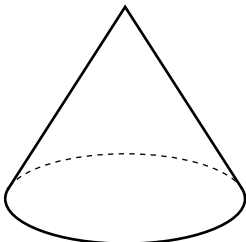
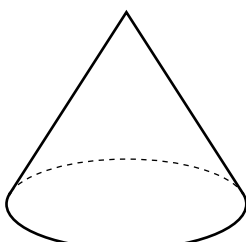
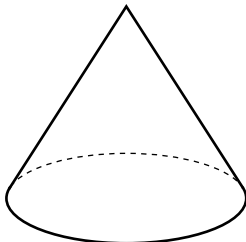
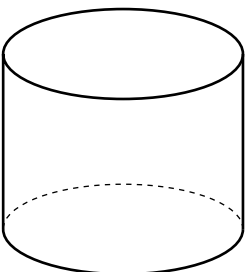
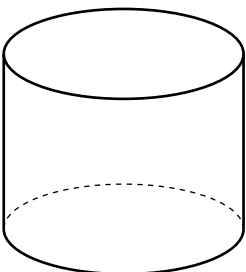
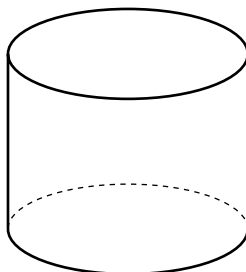
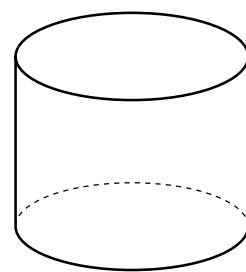


STUDY LINK
11•2

Comparing Geometric Solids



Name the figures, and label their bases, vertices, and edges.

Geometric Solids			
Name	Bases	Vertices	Edges
Example  cube			
1.  _____			
2.  _____			
3.  _____			



LESSON
11•2

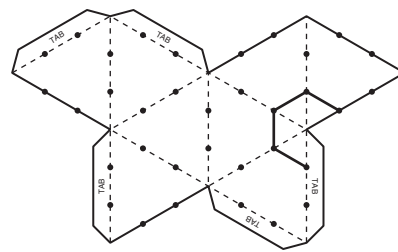
Exploring Truncated Polyhedrons



Truncated polyhedrons are formed by shortening the edges of the solid and cutting off the vertices. Follow the steps below to make models of an octahedron and a truncated octahedron.

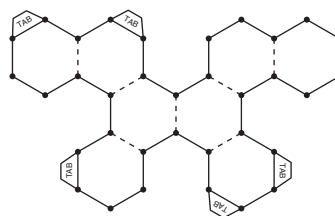
Part 1: Octahedrons

1. Use a centimeter ruler to mark dots on the lines of the pattern on *Math Masters*, page 330 so that the lines are divided into thirds.
2. Use a colored pencil or marker to connect the dots to form triangles around the vertices of the octahedron.
3. Cut out and assemble the octahedron model.
4. Hold the model so that a vertex is facing you. What shape is formed by the colored lines? _____

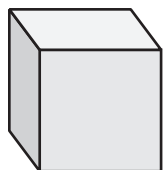


Part 2: Truncated Octahedrons

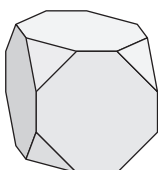
5. Repeat steps 1 and 2 with your second copy of the octahedron pattern.
6. Cut out the pattern. Then cut along the colored lines. You will cut off the vertices and parts of the tabs. Assemble the model.
7. What two shapes are contained in the model?



8. What shapes are contained in a truncated hexahedron?



Hexahedron

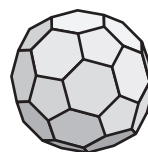


Truncated Hexahedron

9. What shapes are contained in a truncated icosahedron?



Icosahedron



Truncated Icosahedron

STUDY LINK
11•3

Volume of Cylinders



Use these two formulas to solve the problems below.

Formula for the Volume of a Cylinder

$$V = B * h$$

where V is the volume of the cylinder,
 B is the area of the cylinder's base, and
 h is the height of the cylinder.

Formula for the Area of a Circle

$$A = \pi * r^2$$

where A is the area of the circle and r is
the length of the radius of the circle.

1. Find the smallest cylinder in your home. Record its dimensions, and calculate its volume.

radius = _____ height = _____

Area of base = _____ Volume = _____

2. Find the largest cylinder in your home. Record its dimensions, and calculate its volume.

radius = _____ height = _____

Area of base = _____ Volume = _____

3. Write a number model to estimate the volume of:

a. Your toaster _____

b. Your television _____

4. Is the volume of the largest cylinder more or less than the volume of your toaster? _____

About how much more or less? _____

5. Is the volume of the largest cylinder more or less than the volume of your television set? _____

About how much more or less? _____

Practice

6. $6\frac{1}{3} * \frac{2}{5} =$ _____

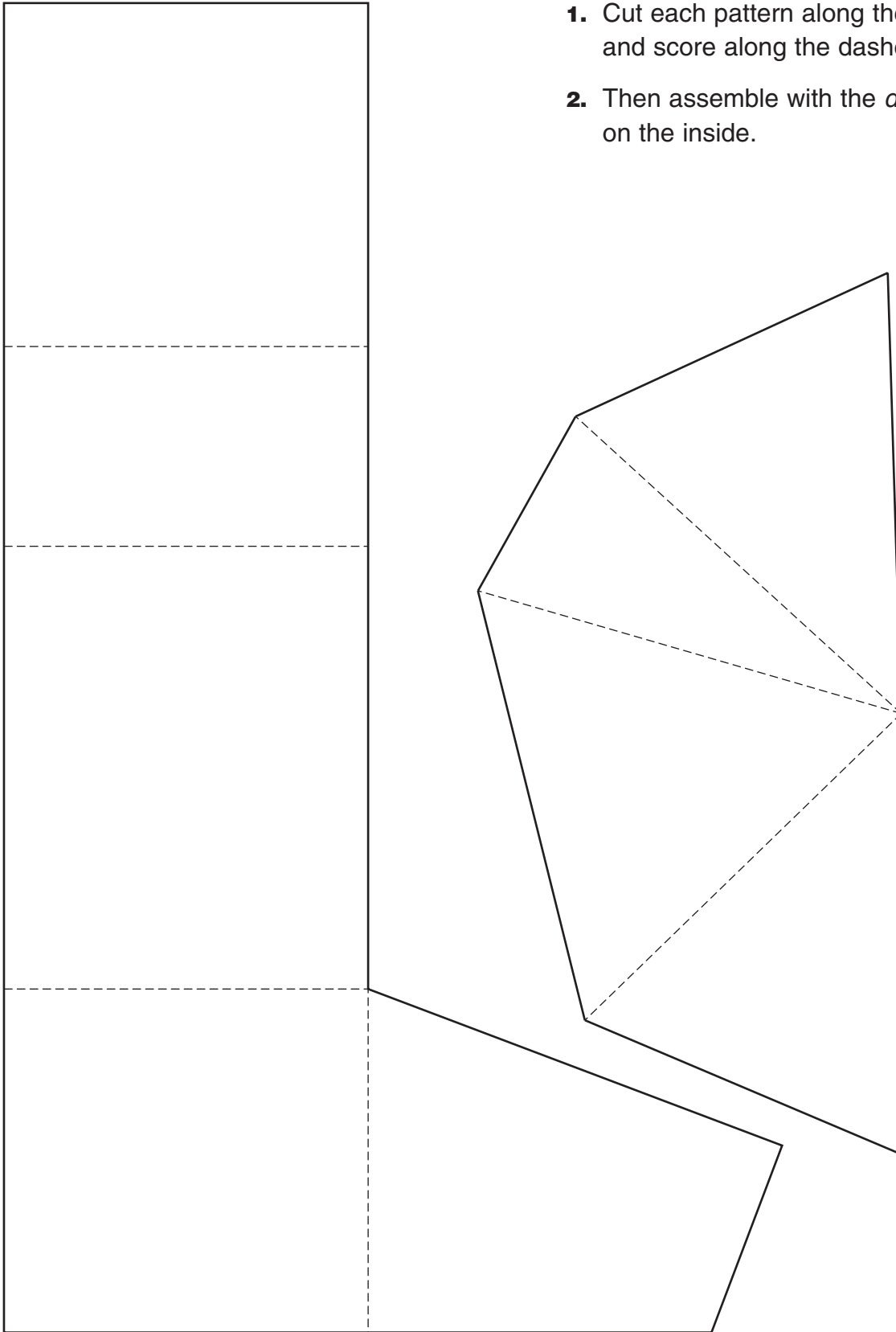
7. $10\frac{6}{8} * \frac{1}{2} =$ _____

8. $4 - 2.685 =$ _____



LESSON
11•4**Prism and Pyramid Patterns**

1. Cut each pattern along the solid lines, and score along the dashed lines.
2. Then assemble with the *dashed lines* on the inside.



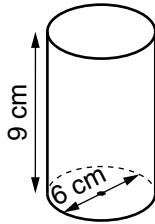
STUDY LINK
11.4

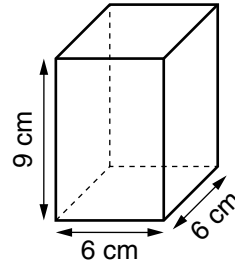
Comparing Volumes



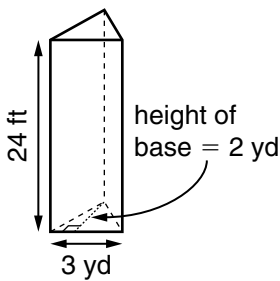
Use $>$, $<$, or $=$ to compare the volumes of the two figures in each problem below.

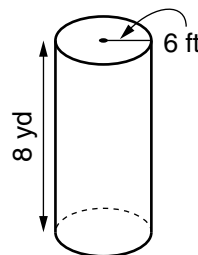
1.



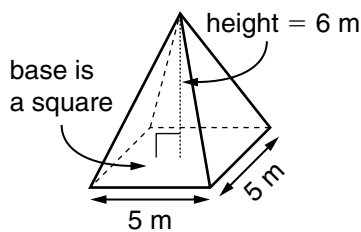


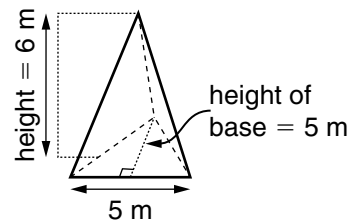
2.





3.





4. Explain how you got your answer for Problem 3.

Practice

5. $4\frac{1}{3} + 2\frac{4}{9} =$ _____

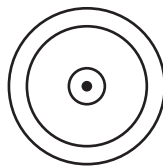
6. $2\frac{6}{7} - 1\frac{1}{3} =$ _____

7. $6 * 10^5 =$ _____

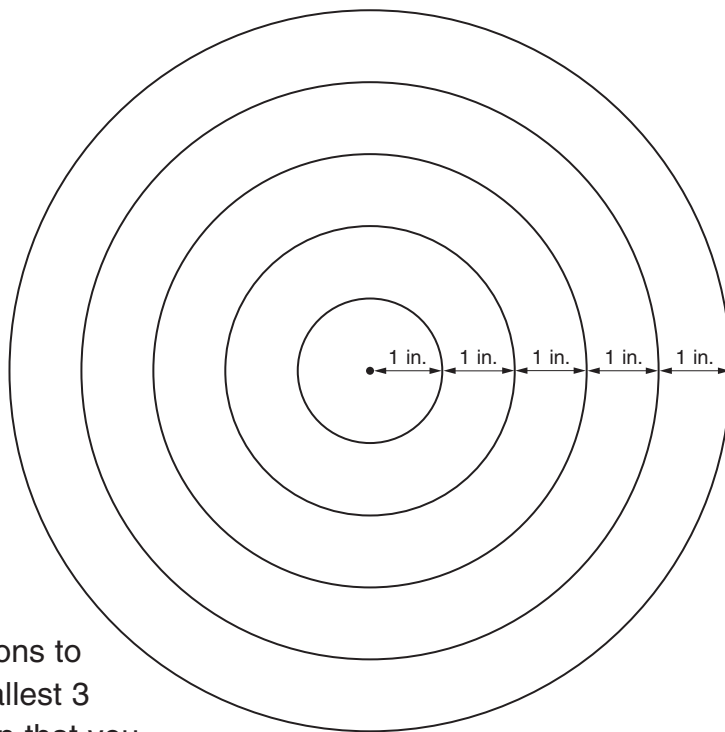
8. $584 \div 23 =$ _____

LESSON
11•4**Finding the Area of Concentric Circles**

Concentric circles are circles that have the same center, but the radius of each circle has a different length.



The smallest of the 5 concentric circles below has a radius of 1 in. The next largest circle has a radius of 2 in. The next has a radius of 3 in. The next has a radius of 4 in., and the largest circle has a radius of 5 in. The distance from the edge of one circle to the next larger circle is 1 in.



1. Use colored pencils or crayons to shade the region of the smallest 3 circles red. Shade the region that you can see of the next circle yellow, and the region that you can see of the largest circle orange.

Which region has the greater area, the red region or the orange region?

2.
 - a. How can you change the distance between the circles to make the area of the yellow region equal to the area of the red region? Explain your answer on the back of this page.
 - b. How can you change the distance between the circles to make the area of the yellow region equal to the area of the orange region? Explain your answer on the back of this page.

STUDY LINK
11•5

A Displacement Experiment



Try this experiment at home.

Materials

- ☐ drinking glass
- ☐ water
- ☐ 2 large handfuls of cotton
(Be sure to use real cotton. Synthetic materials will not work.)

Directions

- ◆ Fill the drinking glass almost to the top with water.
- ◆ Put the cotton, bit by bit, into the glass. Fluff it as you go.

If you are careful, you should be able to fit all of the cotton into the glass without spilling a drop of water.

Think about what you know about displacement and volume. Why do you think you were able to fit the cotton into the glass without the water overflowing?

[illegible]

LESSON
11•5**A Boat and a Stone**

A thought experiment uses the imagination to solve a problem. Mathematicians, physicists, philosophers, and others use thought experiments to investigate ideas about nature and the universe.

One early example of a thought experiment attempts to show that space is infinite. Use your imagination to picture what is being described in the experiment below.

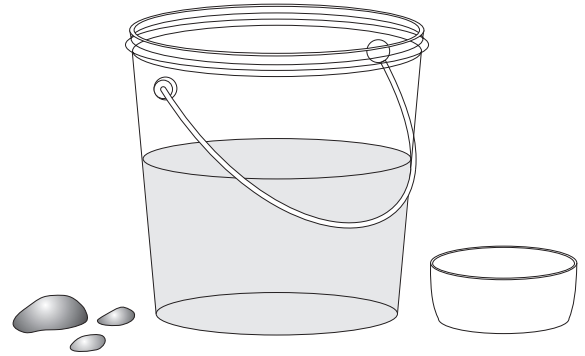
If there is a boundary to the universe, we can toss a spear at it. If the spear flies through, it isn't a boundary after all. If the spear bounces back, then there must be something beyond the supposed edge of space—a cosmic wall which is itself in space that stopped the spear. Either way, there is no edge of the universe; space is infinite.

Often it is impossible to investigate the situation in a thought experiment directly. This might be because of physical or technological limitations. But the thought experiment in Problem 1 can be modeled directly. Solve Problem 1, and then follow the directions in Problem 2 to model the experiment.

1. Imagine that you are in a small boat. There is a large stone in the bottom of the boat. The boat is floating in a swimming pool. If you throw the stone overboard, does the level of the boat on the water go up, down, or stay the same? Does the level of the water in the pool go up, down, or stay the same?

LESSON
11.5**A Boat and a Stone** *continued***2. Model the thought experiment, “A Boat and a Stone.”****Materials**

- ☐ bucket or clear container
- ☐ small container that floats and fits in the bucket or clear container with plenty of space all around
- ☐ several rocks ☐ water
- ☐ waterproof marker

**Directions:**

- a.** Fill the bucket part way up with water. Make sure the water is deep enough to cover the rock.
- b.** Place a rock in the small container, and float it in the bucket. If the small container sinks, try a smaller rock. If the small container tilts over into the water, try a larger rock.
- c.** After the water settles, mark the height of the water on the bucket with the marker. If your bucket is clear, mark the outside. If not, mark the inside wall. Also, mark the height of the water on the outside of the small container.
- d.** Take the rock out of the small container, and gently drop it into the water.
- e.** Describe the changes in the height of the water on the outside of the small container.

- f.** Describe the changes in the height of the water in the bucket.

- g.** Do the changes agree with your thought experiment solutions? Why or why not?

STUDY LINK
11•6

Units of Volume and Capacity



Write $>$, $<$, or $=$ to compare the measurements below.

1. 5 cups _____ 1 quart 2. 30 mL _____ 30 cm³ 3. 1 quart _____ 1 liter
4. 15 pints _____ 8 quarts 5. 100 cm³ _____ 1 gallon 6. 10 cups _____ 5 pints

Circle the unit you would use to measure each of the following.

7. The volume of a square pyramid

gallons cubic inches ounces meters

8. The amount of milk a fifth grader drinks in a week

gallons milliliters ounces meters

9. The amount of water used to fill a swimming pool

gallons milliliters ounces meters

10. The amount of penicillin given in a shot

gallons milliliters liters meters

11. The volume of a rectangular prism

gallons cubic centimeters liters meters

12. Would you think of volume or capacity if you wanted to know how much juice a jug holds? _____

13. Would you think of volume or capacity if you wanted to know how much closet space a stack of boxes would take up? _____

Practice

14. $-200 + (-50) =$ _____ 15. $685 * 201 =$ _____

16. $13\frac{1}{5} - 2\frac{4}{5} =$ _____ 17. $3.84 \div 8 =$ _____

STUDY LINK
11.7

Volume and Surface Area


Area of rectangle:

$$A = l * w$$

Circumference of circle:

$$C = \pi * d$$

Volume of rectangular prism:

$$V = l * w * h$$

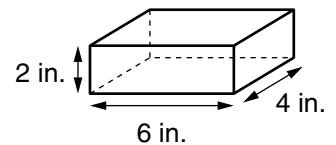
Area of circle:

$$A = \pi * r^2$$

Volume of cylinder:

$$V = \pi * r^2 * h$$

1. Kesia wants to give her best friend a box of chocolates. Figure out the least number of square inches of wrapping paper Kesia needs to wrap the box. (To simplify the problem, assume that she will cover the box completely with no overlaps.)



Amount of paper needed: _____

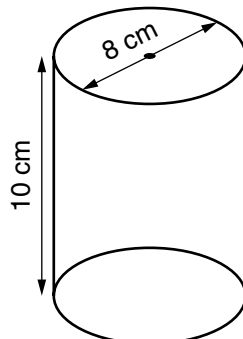
Explain how you found the answer.

2. Could Kesia use the same amount of wrapping paper to cover a box with a larger volume than the box in Problem 1? _____ Explain.

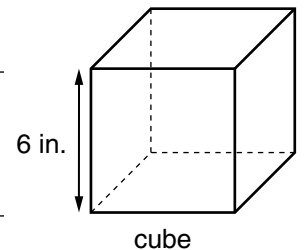
Find the volume and the surface area of the two figures in Problems 3 and 4.

3. Volume:

Surface area:


4. Volume:

Surface area:



LESSON

11•7

A Surface-Area Investigation



In each problem below, the volume of a rectangular prism is given. Your task is to find the dimensions of the rectangular prism (with the given volume) that has the smallest surface area. To help you, use centimeter cubes to build as many different prisms as possible having the given volume.

Record the dimensions and surface area of each prism you build in the table. Do not record different prisms with the same surface area. Put a star next to the prism with the smallest surface area.

1.

Dimensions (cm)	Surface Area (cm ²)	Volume (cm ³)
2 × 6 × 1	40	12
		12
		12
		12

2.

Dimensions (cm)	Surface Area (cm ²)	Volume (cm ³)
		24
		24
		24
		24
		24
		24

3.

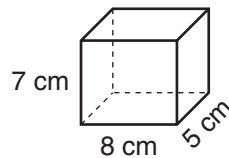
If the volume of a prism is 36 cm³, predict the dimensions that will result in the smallest surface area. Explain.

4.

Describe a general rule for finding the surface area of a rectangular prism in words or with a number sentence.

LESSON
11·7**Volume and Surface Area of Solids****Area of rectangle:** $A = l * w$ **Circumference of circle:** $C = \pi * d$ **Area of triangle:** $A = \frac{1}{2} * b * h$ **Area of circle:** $A = \pi * r^2$ **Volume of rectangular prism:** $V = B * h$ **Volume of cylinder:** $V = \pi * r^2 * h$

Record the dimensions, and find the surface area and volume for each figure below.

1. Rectangular prism

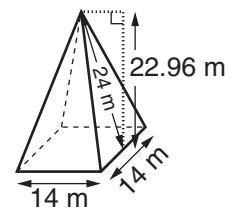
length of base = _____

width of base = _____

height of prism = _____

Volume = _____

Surface Area = _____

2. Square pyramid

length of base = _____

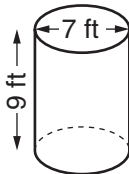
width of base = _____

height of pyramid = _____

slant height = _____

Volume = _____

Surface Area = _____

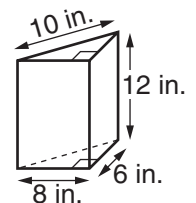
3. Cylinder

diameter = _____

height = _____

Volume = _____

Surface Area = _____

4. Triangular prism

length of base = _____

height of base = _____

length of hypotenuse = _____

height of prism = _____

Volume = _____

Surface Area = _____

Reminder: The hypotenuse is the side of a right triangle opposite the right angle.



Probability, Ratios, and Rates

A **ratio** is a comparison of two quantities with the same unit. For example, if one house has a floor area of 2,000 ft², and a second house has a floor area of 3,000 ft², the ratio of the areas is 2,000 to 3,000, or 2 to 3, simplified.

To prepare students for working with ratios in algebra, the class will review the meanings and forms of ratios and will solve number stories involving ratios of part of a set to the whole set. Your child will find, write, and solve many number models (equations) for ratio problems.

Your child will continue to use the American Tour section of the *Student Reference Book* as part of the discussion of ratios. We will also be doing projects based on information in the American Tour.

A **rate** is a comparison of two quantities with different units. For example, speed is expressed in miles per hour. In our study of rates, students will determine their own heart rates (heartbeats per minute). Then they will observe the effect of exercise on heart rate and represent the class results graphically.

We will continue our study of probability by looking at situations in which a sequence of choices is made. For example, if a menu offers you 2 choices of appetizer, 4 choices of entrée, and 3 choices of dessert, and you choose one of each kind, there are $2 * 4 * 3$ or 24 different possible combinations for your meal. If all the choices were equally appealing (which is unlikely), and you chose at random, the probability of any one combination would be $\frac{1}{24}$.

Your child will play *Frac-Tac-Toe*, which was introduced in Unit 5, as well as a new game, *Spoon Scramble*, to practice operations and equivalencies with fractions, decimals, and percents.

You can help your child by asking questions about homework problems; by pointing out fractions, percents, and ratios that you encounter in everyday life; and by playing *Frac-Tac-Toe* and *Spoon Scramble* to sharpen his or her skills.



Please keep this Family Letter for reference as your child works through Unit 12.

Vocabulary

Important terms in Unit 12:

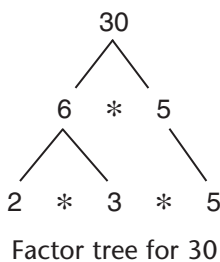
common factor Any number that is a factor of two or more counting numbers. The common factors of 18 and 24 are 1, 2, 3, and 6.

equally likely outcomes Outcomes of a chance experiment or situation that have the same probability of happening. If all the possible outcomes are equally likely, then the probability of an event is equal to: $\frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$

factor tree A method used to obtain the prime factorization of a number. The original number is written as a product of factors.

Then each of these factors is written as a product of factors, and so on, until the factors are all prime numbers. A factor tree

looks like an upside down tree with the root (the original number) at the top, and the leaves (the factors) beneath it.



greatest common factor The largest factor that two or more counting numbers have in common. For example, the common factors of 24 and 36 are 1, 2, 3, 4, 6, and 12. Thus, the greatest common factor of 24 and 36 is 12.

least common multiple The smallest number that is a multiple of two or more numbers. For example, while some common multiples of 6 and 8 are 24, 48, and 72, the least common multiple of 6 and 8 is 24.

multiplication counting principle A way of determining the total number of possible outcomes for two or more separate choices. Suppose, for example, you roll a die and then flip a coin. There are 6 choices for which number on the die lands up and 2 choices for which side of the coin shows. Then there are $6 * 2$, or 12 possible outcomes all together: (1,H), (1,T), (2,H), (2,T), (3,H), (3,T), (4,H), (4,T), (5,H), (5,T), (6,H), (6,T).

prime factorization A counting number expressed as a product of prime number factors. For example, the prime factorization of 24 is $2 * 2 * 2 * 3$, or $2^3 * 3$.

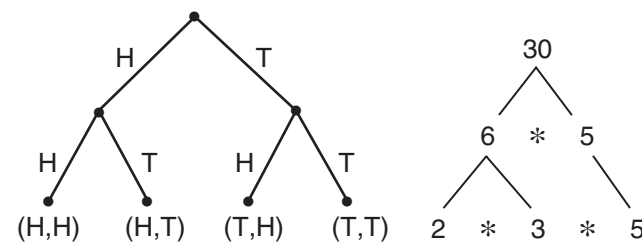
probability A number from 0 to 1 that tells the chance that an event will happen. For example, the probability that a fair coin will show heads is $\frac{1}{2}$. The closer a probability is to 1, the more likely it is that the event will happen. The closer a probability is to 0, the less likely it is that the event will happen.

rate A comparison by division of two quantities with unlike units. For example, traveling 100 miles in 2 hours is an average rate of 100 mi/2 hr, or 50 miles per hour. In this case, the rate compares distance (miles) to time (hours).

ratio A comparison by division of two quantities with the same units. Ratios can be fractions, decimals, percents, or stated in words. Ratios can also be written with a colon between the two numbers being compared. For example, if a team wins 3 out of 5 games played, the ratio of wins to total games can be written as $\frac{3}{5}$, 3/5, 0.6, 60%, 3 to 5, or 3:5 (read “three to five”).

tree diagram A network of points connected by line segments and containing no closed loops. Factor trees are tree diagrams used to factor numbers. Probability trees are tree diagrams used to represent probability situations in which there is a series of events.

The first tree diagram below represents flipping one coin two times. The second tree diagram below shows the prime factorization of 30.



Tree diagrams

Do-Anytime Activities

To work with your child on the concepts taught in this unit and in previous units, try these interesting and rewarding activities:

1. Identify different ratios, and ask your child to write each ratio using words, a fraction, a decimal, a percent, and a colon. For example, the ratio of 1 adult for every 5 students could be written as 1 to 5, $\frac{1}{5}$, 0.2, 20%, or 1:5.
2. Play one of the games in this unit with your child: *Frac-Tac-Toe*, *Name That Number*, or *Spoon Scramble*.
3. Read the book *Jumanji* with your child, and review the possible outcomes when rolling two dice. Ask your child to verify the probabilities of rolling certain number combinations by recording the outcomes for 100 rolls of a pair of dice.
4. Identify rate situations in everyday life, and ask your child to solve problems involving rates. For example, find the number of miles your car travels for each gallon of gas, or find the number of calories that are burned each hour or minute for different types of sports activities.

Building Skills through Games

In Unit 12, your child will practice skills with probability, ratios, and rates by playing the following games. For detailed instructions, see the *Student Reference Book*.

Frac-Tac-Toe See *Student Reference Book*, pages 309–311. This is a game for two players. Game materials include 4 each of the number cards 0–10, pennies or counters of two colors, a calculator, and a gameboard. The gameboard is a 5-by-5 number grid that resembles a bingo card. Several versions of the gameboard are shown in the *Student Reference Book*. *Frac-Tac-Toe* provides students with practice in converting fractions to decimals and percents.

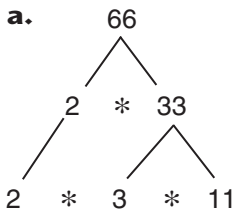
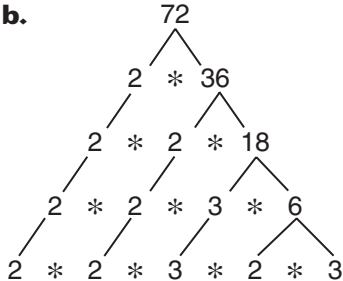
Name That Number See *Student Reference Book*, page 325. This is a game for two or three players. Game materials include the Everything Math Deck or a complete deck of number cards. Playing *Name That Number* provides students with practice in working with operations and in using the order of operations.

Spoon Scramble See *Student Reference Book*, page 330. This is a game for four players using 3 spoons and a deck of 16 *Spoon Scramble Cards*. *Spoon Scramble* provides students with practice identifying equivalent expressions for finding a fraction, a decimal, or a percent of a number.

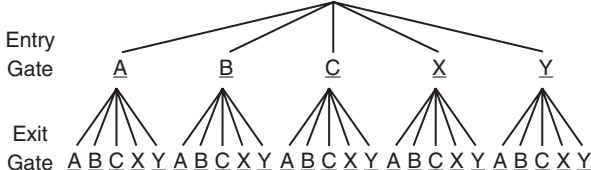
As You Help Your Child with Homework

As your child brings assignments home, you might want to go over the instructions together, clarifying them as necessary. The answers listed below will guide you through this unit's Study Links.

Study Link 12•1

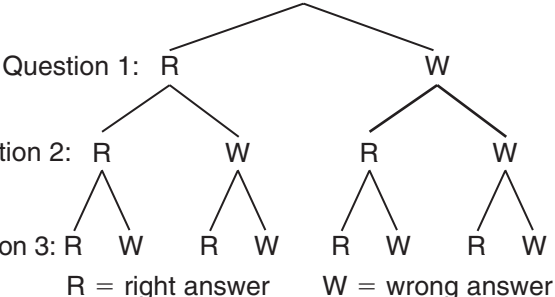
- 1. a.**  **b.** 
- 2. a.** $\frac{10}{33}$ **b.** $\frac{11}{12}$ **c.** $\frac{5}{18}$
- 3.** $250 = 5 * 5 * 5 * 2$
- 4. a.** 32 **b.** 49 **5.** $\frac{2}{3}$

Study Link 12•2

- 1.** $5 * 5 = 25$
- 2.** 

3. No; Sample answer: Some gates will probably be used more than other gates.

4. 20

- 5. a.** 
- R = right answer W = wrong answer

b. $\frac{1}{8}$

Study Link 12•3

- 1.** Sixteen out of twenty-five
- 2.** $\frac{16}{25}$ **3.** 64% **4.** 16:25
- 5.** 23:50; 0.46 of the cars were blue

- 6.** $\frac{2}{3}$; 6:9; $66\frac{2}{3}\%$ of the people were swimmers
- 7.** 7 out of 8; 35:40 of the caps sold were baseball caps

Study Link 12•4

- 1. a.** 4 **b.** 16 **2.** 15
- 3.** 16 **4.** 8 **5.** 32
- 6.** 98 R38 **7.** 9,016 **8.** 90.54

Study Link 12•5

- 1.** 8 **2.** 24 **3.** 45
- 4.** 60 **5.** 20 **6.** 26

7. $\frac{2}{5} = \frac{\boxed{46}}{115}$; 46 students

9. $\frac{1.50}{3} = \frac{\boxed{45.00}}{90}$; \$45.00

- 11.** 216 **12.** 729

Study Link 12•6

1. a.

Number of Spiders	27,000	54,000	81,000	108,000	135,000
Pounds of Spider Web	1	2	3	4	5

b. 270,000

- 3.** 1,000 **4.** 930 **5.** $7\frac{1}{2}$, or 7.5

Study Link 12•7

- 1.** $3\frac{3}{4}$ in. **3.** $1\frac{3}{4}$ lb **5.** $20\frac{7}{8}$ in.
- 7.** $50\frac{2}{5}$ kg **9.** 34 **11.** 180

Study Link 12•8

- 2.** 8 lunches
- 4. a.** 1 to 1 **b.** 26 to 104, or $\frac{1}{4}$ **c.** 8 to 16, or $\frac{1}{2}$
- 5.** $3\frac{4}{7}$ **6.** 5 **7.** 12.5 **8.** 8